[illegible]

In this paper, we restrict attention to matrix cracking which accompanies the complete fracture of an isolated fiber. It is assumed that this fracture configuration results in a defect which can be modeled as a penny-shaped crack located in a single

$$\sigma_{\theta\theta}^{(m)}(r, z) = -\frac{A_m}{r^2} + 2B_m \qquad (1b)$$

In addition, the displacement and stress fields in the two domains [illegible] (m) [illegible]

$$\eta R_3(s,t) = \sinh(st)(a_7a_{15} - a_{10}a_{12}) - (a_{15}I_0 + a_{10}I_1)\{\sinh(st)(2\nu_f - 1) + st\cosh(st)\} \quad (51)$$

$$\eta R_4(s,t) = \sinh(st)(a_7a_{16} - a_{11}a_{12}) - (a_{16}I_0 + a_{11}I_1)\{\sinh(st)(2\nu_f - 1) + st\cosh(st)\}. \quad (52)$$

Considering the integrals of Eqs. (32) to (35) containing $F_1(u)$, the integral expressions (36) to (39) and the representation (43) it can be shown that

$$-\frac{2}{\pi}\int_0^\infty \{2(1-\nu_f)f_1 + uf_2\}\frac{F_1(u)}{u}J_0(ua)du$$

$$= -\frac{2}{\pi}\int_0^a [(3-2\nu_f)\sinh(su)K_0(sa) - saK_1(sa)\sinh(su) + su\cosh(su)K_0(sa)]m(u)du;$$

$$+ sI_1(sa)\left[\int_a^b uh(u)e^{-su}du + \int_b^\infty ug(u)e^{-su}du\right];$$

$$0 < s < \infty \quad (58)$$

$$-\frac{2}{\pi}a\mu_m\int_0^\infty \left[(-f_3 + uf_4)J_0(ua) + \{(1-2\nu_m)f_3 - uf_4\}\frac{J_1(ua)}{ua}\right]F_2(u)du$$

$$= \mu_m\left[\int_a^b h(t)e^{-st}\left[sa\left\{\frac{1}{2}I_0(sa) + saI_1(sa) - stI_0(sa)\right\}\right.\right.$$

$$M(t) = \frac{\pi \mu_m}{t} m(t) \quad (64)$$

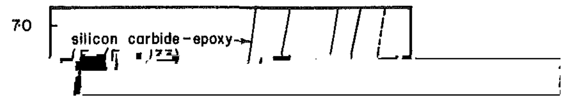
and the kernel functions S_1 and S_2 are given by

$$S_1(u, t) = \int_0^\infty e^{-su}[\{(3 - 2\nu_m)I_0(sa) + saI_1(sa)$$

$$- s^2 u I_1(sa)\Big\} R_4(s, t)\Big] ds. \quad (68)$$

$$S_4(u, t) = -\int^\infty [\{(3 - 2\nu_f) \sinh (su) K_0(sa)$$

and subjected to a uniform strain ϵ_0 (or uniform stress $E_m\epsilon_0$) (Sneddon, 1946), i.e.,



$$a_1 = s^2a^2(I_1^2 - I_0^2) + 2(1 - \nu_f)I_1^2 \quad \text{(A7)}$$

$$a_2 = sa\Gamma\{K_0I_1 + K_1I_0\} \quad \text{(A8)}$$

$$a_3 = -\Gamma[K_1I_1\{2 - 2\nu_m + s^2a^2\} + 2(1 - \nu_m)saI_0K_1 + 2sa(1 - \nu_m)K_0I_1 + K_0I_0s^2a^2] \quad \text{(A9)}$$

$$a_4 = sa(I_1^2 - I_0^2) + 4(1 - \nu_f)I_0I_1 \quad \text{(A10)}$$

[redacted]

$$a_6 = -\{sa(I_1K_1 + I_0K_0) + 4(1 - \nu_m)I_0K_1\} \quad \text{(A12)}$$

$$a_7 = saI_1 + \left\{ saK_1(a_2a_4 - a_5a_1) + K_0(a_3a_4 - a_6a_1) \right\} \quad \text{(A13)}$$

[redacted]

$$a_8 = 1 + \frac{I_1(a_2saK_1 + a_3K_0)}{(a_2a_6 - a_3a_5)} \quad \text{(A14)}$$

$$a_9 = \frac{I_0(saa_2K_1 + a_3K_0)}{(a_2a_6 - a_3a_5)} \quad \text{(A15)}$$

$$I_1(a_3K_0 + saa_2K_1)$$

[redacted]

$$X_1(s) = [(3 - 2\nu_m)I_0(sa) + saI_1(sa)]\left[\int_a^b h(u)e^{-su}du + \int_b^\infty g(u)e^{-su}du\right] - sI_0(sa)\left[\int_a^b uh(u)e^{-su}du + \int_b^\infty ug(u)e^{-su}du\right]$$

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$$+ su\cosh(su)K_0(sa)]m(u)du \quad \text{(A24)}$$

$$X_2(s) = -\left\{\frac{sa}{2}[I_0(sa) + I_2(sa)] + 2\nu_mI_1(sa)\right\}$$

[redacted]

$$\times \left\{\int_a^b h(u)e^{-su}du + \int_b^\infty g(u)e^{-su}du\right\} + sI_1(sa)\left[\int_a^b uh(u)e^{-su}du + \int_b^\infty ug(u)e^{-su}du\right]$$

$$\frac{2}{\,}\int^a$$

[redacted]

The expressions for $P_i(s, t)$ $(i = 1, 2, 3, 4)$ occurring in the kernel functions S_1 and S_2 defined by Eqs. (65) and (66) are given by

$$\zeta P_1(s, t) = -[\{(a_{12}I_0 + a_7 I_1)a_3 I_1 + (a_8 I_1 + a_{13}I_0)$$

[redacted]

$$- (a_2 a_4 - a_1 a_5)(a_{13}I_0 + a_8 I_1)\}$$

$$\times \{2\nu_m L_1(s, t) - sL_2(s, t)\}] \quad \text{(A28)}$$

$$\zeta P_2(s, t) = -[\{(a_{12}I_0 + a_7 I_1)a_3 I_0 + (a_9 I_1 + a_{14}I_0)$$

[redacted]

$$- (a_2 a_4 - a_1 a_5)(a_{14}I_0 + a_9 I_1)\}$$

where the unknown functions $X(t)$ are defined as

$$\mathbf{X}(t) = \begin{cases} M(t); & 0 \le t < a \\ H(t); & a \le t < b \end{cases} \quad \text{(B2)}$$

[redacted]

The right-hand side of the prescribed function $\mathbf{B}(t)$ in (B1) takes the form

$$\mathbf{B}(t) = \begin{cases} t\Gamma\Omega; & 0 \le t < a \\ (b^2 - t^2)^{1/2}; & a \le t < b \\ 0; & b \le t < \infty \end{cases} \quad \text{(B3)}$$

[redacted]

where $N = N_1 + N_2 + N_3$. Using the above procedures and representations, the discretized form of (B1) can be written as

$$\mathbf{A}_n \mathbf{X}_n = \mathbf{B}_n \qquad \text{(B11)}$$

$$\mathbf{A}_{ij} = \delta_{ij} + K(t_i, t_j)\Delta t, \qquad \text{(B14)}$$

where δ_{ij} is the Kronecker delta and Δt is given as h_x, h_y or