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Enhanced consolidation in brittle geomaterials susceptible to damage

A. T. Mahyari and A. P. S. Selvadurai*

Department of Civil Engineering and Applied Mechanics, McGill University, 817 Sherbrooke Street West, Montreal, Q.C., Canada H3A 2K6

SUMMARY

This paper examines consolidation behaviour of saturated geomaterials with a matrix component which is susceptible to damage. Finite-element-based computational model accounts for the alteration in both the deformability and permeability characteristics of the porous material due to damage evolution. The isotropic damage criteria governing the evolution of elastic stiffness and hydraulic conductivity parameters are characterized by the dependency of the damage variable on the distortional strain invariant. The computational procedure is utilized to evaluate the extent to which the time-dependent axisymmetric indentation behaviour of a rigid circular punch on a poroelastic half-space can be influenced by the damage evolution in the porous skeleton. © 1998 John Wiley & Sons, Ltd.

KEY WORDS: poroelasticity; isotropic damage; brittle geomaterials; enhanced consolidation; saturated geomaterials; computational modelling; indentation of geomaterials

1. INTRODUCTION

The classical theory of poroelasticity developed by Biot[1, 2] examines the coupled behaviour of fluid flow and elastic deformations on the consolidation process of porous materials saturated with either incompressible or compressible pore fluids. The theory of poroelasticity has been successfully applied to examine time-dependent transient phenomena in a variety of natural and synthetic materials, including geomaterials and biomaterials.[3] The assumption of linear elastic behaviour of the porous skeleton is a significant limitation in the application of the classical theory of poroelasticity to brittle geomaterials which could exhibit stiffness changes and, in particular, elastic stiffness degradation in the constitutive behaviour of the skeleton of the geomaterial. This non-linear behaviour can be due to development of microcracks and microvoids in the porous fabric of the geomaterial which essentially retains its elastic nature (i.e. absence of irreversible plasticity phenomena). Such damage or microvoid and microcrack evolution can result in the alteration of the permeability characteristics of the porous medium. The effect of such [illegible] on either the degradation of elastic moduli, and in extreme situations,
[illegible]

on concrete. The theory of damage mechanics has been extensively applied to model the behaviour of such brittle geomaterials.[7–10] The effect of microcrack developments on permeability characteristics of saturated geomaterials has also been observed by Zoback and Byerlee,[11] Shiping [illegible].,[12] and Kiyama [illegible].[13] in rocks and by Samaha and Hover[14] in concrete.

When considering saturated poroelastic geomaterials, their consolidation response can be influenced by the evolution of damage in the porous skeleton. The notion of continuum damage is considered to be more relevant to geomaterials such as soft rocks and overconsolidated clays were progressive softening in an [illegible] sense can occur due to generation of microvoids or microcracks. The classical theory of continuum damage mechanics[15] can be extended to model such damage phenomena in porous saturated materials. This theory simulates the effect of microcrack developments on the behaviour of materials prior to the development of [illegible] (i.e. fractures). In such modelling, damage is interpreted as a reduction in the stiffness of the material due to the generation of microcracks and other microdefects. In this study attention is restricted primarily to [illegible] of material where the associated damage processes before inception of strain softening (i.e. strictly pre-peak elastic behaviour) can be described by an isotropic damage model. Admittedly, the damage processes are expected to be highly anisotropic in nature and could invariably be restricted to localized zones. The effect of soil skeletal damage on the consolidation behaviour of saturated geomaterials can be examined by representing the stiffness properties and the permeability characteristics of porous medium as a function of the state of damage in the saturated geomaterial. Cheng and Dusseault[16] developed an anisotropic damage model to examine the poroelastic behaviour of saturated geomaterials. Their studies were, however restricted to the case where there was no corresponding evolution in the permeability characteristics of the geomaterials during the damage processes.

In this study a finite element technique is used to examine the influence of damage-induced alterations in both the elastic stiffness and the permeability characteristics of the porous geomaterial on the corresponding consolidation response of a saturated poroelastic medium. The isotropic damage evolution law used in the analysis is characterized by the dependency of damage parameters on the distortional strain invariant. Two different phenomenological damage criteria governing the evolution of permeability characteristics are postulated from experimental observations on saturated geomaterials. Finally, the numerical procedure is utilized to evaluate the extent to which the time-dependent indentation behaviour of a smooth rigid circular punch with a permeable base resting on a poroelastic half-space can be influenced by the damage evolution in the porous skeleton (Figure 1).

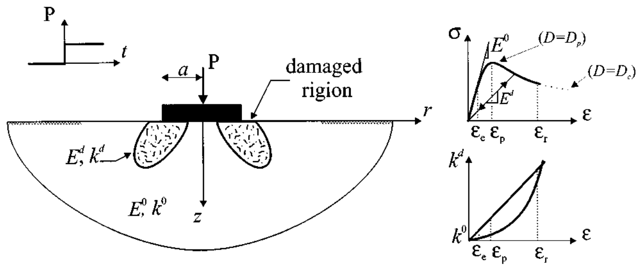

Figure 1. Indentation of a brittle geomaterial

2. GOVERNING EQUATIONS

The basic equations governing Biot's theory of poroelasticity are summarized for completeness. The constitutive equations governing the quasi-static response of a poroelastic medium, which consists of a porous isotropic elastic soil skeleton saturated with a [illegible] take the forms

$$\sigma_{ij} \text{"} \frac{2\mu v}{1 \,!\, 2v}\varepsilon_{kk}\delta_{ij} \,\#\, 2\mu\varepsilon_{ij} \,!\, \frac{3(v_{\mathrm{u}} \,!\, v)}{(1 \,!\, 2v)(1 \,\#\, v_{\mathrm{u}})} \delta_{ij} \tag{1a}$$

$$\text{"} \frac{2\mu \quad {}^{2}(1 \,!\, 2v)(1 \,\#\, v_{\mathrm{u}})^{2}}{9(v_{\mathrm{u}} \,!\, v)(1 \,!\, 2v_{\mathrm{u}})}\zeta_{\upsilon} \,!\, 2\mu \quad (1 \,\#\, v_{\mathrm{u}})$$

3. FINITE ELEMENT FORMULATIONS

Finite element methods have been widely applied for the study of problems in poroelasticity (see e.g. References 18. 20). Reviews of both analytical and numerical approaches to the study of soil consolidation related to poroelastic media are given by Lewis and Schreßer developments, the theory of continuum damage mechanics has been widely used to predict the that damage evolution is a function of the shear strain energy and proposed the following damage evolution equation for rocks:

$$\frac{\partial \Omega}{\partial \xi_{\mathrm{d}}} = \eta \frac{\gamma \xi_{\mathrm{d}}}{1 + \gamma \xi_{\mathrm{d}}} \left(1 - \frac{\Omega}{\Omega_{\mathrm{c}}} \right) \tag{9}$$

where the equivalent shear strain ξ_{d} is defined as

$$\xi_{\mathrm{d}} = (e_{ij} e_{ij})^{1/2}, \quad e_{ij} = \varepsilon_{ij} - \tfrac{1}{3} \varepsilon_{kk} \delta_{ij} \tag{10}$$

and η, γ are material constants which are positive. In this formulation, the normalizing damage measure is the critical damage Ω_{c} which is associated with the damage corresponding to a residual value of the strength of the geomaterial under uniaxial compression. We note that Ω_{c} need not be the only normalizing variable; the formulation can be presented in terms of Ω_{p} the damage at peak loads which can limit the development of localization effects that can result when $\Omega \geq \Omega_{\mathrm{c}}$.

For saturated geomaterials susceptible to damage, the elastic properties and permeability characteristics can alter due to development of microcracks in the porous fabric.

4.1. *Deformability characteristics*

The constitutive parameters applicable to an isotropic poroelastic material which experiences micromechanical damage in the porous fabric can be represented as a function of intact elastic properties by invoking the hypothesis of strain equivalence.[31] The generalized constitutive tensor applicable to damaged materials which exhibit isotropic damage takes the form

$$C^{\mathrm{d}}_{ijkl} = (1 - \Omega) C_{ijkl} \quad (\; = 1, \ldots, 4, \quad \; = 1, \ldots, 4) \tag{11}$$

where C_{ijkl} is the elasticity tensor applicable to virgin elastic materials (see e.g. Reference 33). The damage evolution law can specify the variation of the damage variable (Ω) with the state of strain in material. The damage evolution law proposed by Cheng and Dusseault[16] is employed in this study to model the elastic stiffness degradation of materials. The evolution of damage variable can be obtained by the integration of (9) (between the limits Ω_0 to Ω) as follows:

$$\Omega = \Omega_{\mathrm{c}} - (\Omega_{\mathrm{c}} - \Omega_0)(1 + \gamma \xi_{\mathrm{d}})^{\eta/\gamma D_{\mathrm{c}}} \exp(-\eta \xi_{\mathrm{d}} / \Omega_{\mathrm{c}}) \tag{12}$$

where Ω_0 is the initial value of damage variable corresponding to intact state of material (e.g. $\Omega_0 = 0$ for virgin state of materials).

4.2. *Hydraulic conductivity characteristics*

Development of damage criteria which can account for alterations in the hydraulic conductivity during evolution of damage in saturated geomaterials is necessary for computational modelling of such phenomena in poroelastic media. Literature on the coupling between microcrack developments and permeability evolution in saturated geomaterials is primarily restricted to experimental observations. The effect of microcrack development on the permeability characteristics of fluid saturated geomaterial was first investigated by Zoback and

Figure 3. Permeability evolution in saturated geomaterials (a) after Zoback and Byerlee;[11] and (b) after Shiping .[12]

Byerlee,[11] who conducted triaxial tests on granite. Figure 3 illustrates their experimental observations which indicate that the permeability coefficient is first reduced slightly due to closure of pre-existing microcracks and void channels as a result of the elastic deformations of the intact material, and begins to increase as a result of the growth of existing microcracks and the nucleation of new microcracks. Shiping .[12] examined the permeability evolution of sandstone for a series of complete triaxial stress–strain paths (Figure 3). They observed that permeability characteristics of material can increase nearly by one order of magnitude, prior to the peak values of the stress and can increase up to two orders of magnitude in the strain-softening regime where microcracks tend to localize in shear faults. Kiyama .[13] also observed similar results for permeability evolution of granites in triaxial experiments. This would suggest that localization phenomena can result in significant changes in the permeability in the localization zones. It must be emphasized that in this study the process of localization is excluded from the analysis and all changes in permeability are assumed to materialize in a distributed fashion at stress states well below those which can initiate localization.

There has been only limited work in the context of constitutive modelling of permeability characteristics in damaged porous materials. In this study two phenomenological constitutive models based on the experimental observations by Zoback and Byerlee[11] and Shiping .[12] are proposed for the permeability evolution criteria in poroelastic media. The slight reduction in the hydraulic conductivity of saturated geomaterials in the elastic range prior to the onset of microcrack developments is neglected. The hydraulic conductivity ( ) is assumed to have either linear or quadratic variations with respect to equivalent shear strain ξ_{d} (equation (10)) as follows:

$$ {}^{d} \; '' \; (\; {}_{1} $$

5. COMPUTATIONAL PROCEDURES AND NUMERICAL RESULTS

The effect of soil skeletal damage on the time-dependent poroelastic behaviour of saturated

As an example of the application of the modelling concepts and computational developments, we consider the axisymmetric indentation of a poroelastic half-space susceptible to damage by a rigid porous smooth circular indentor. The poroelastic medium is subjected to a total load $_{/}$ with a time variation in form of a Heaviside step function (Figure 1). The soil skeleton has a Poisson's ratio *v* and the pore fluid within the geomaterial is assumed to be

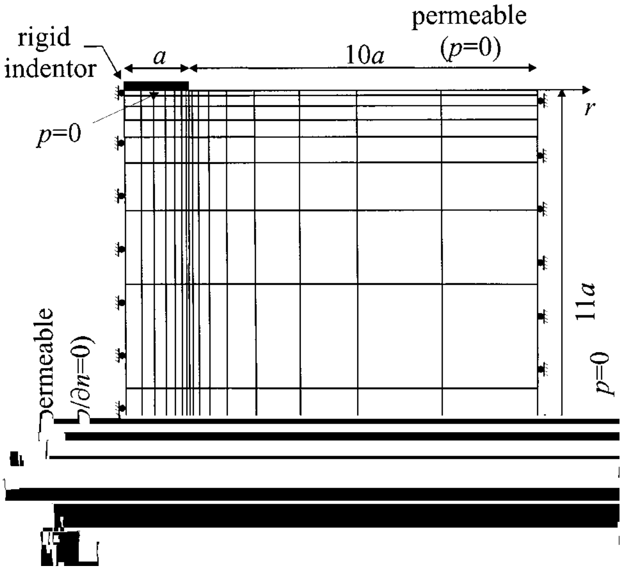

Figure 5. Finite element discretizations

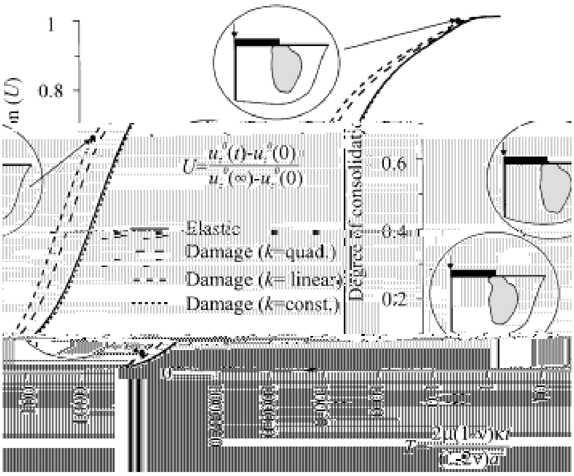

Figure 6. Degree of consolidation and evolution of damaged zone (where , /, $_c$ ' 0·05) with time

conductivity is maximum. Figure 6 also indicates that much of the damage generation takes place instantaneously and further pore pressure diffusion which results in the change of effective stresses, does not appreciably alter the extent of damage. Admittedly no generalizations can be made of this observation since the rate of load transfer will depend on the prescribed damage evolution laws.

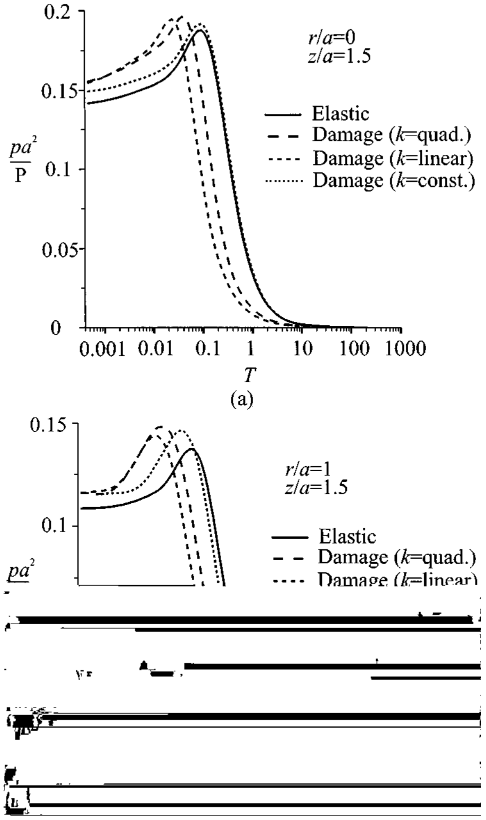

Figure 7. Pore pressure evolution at different depths

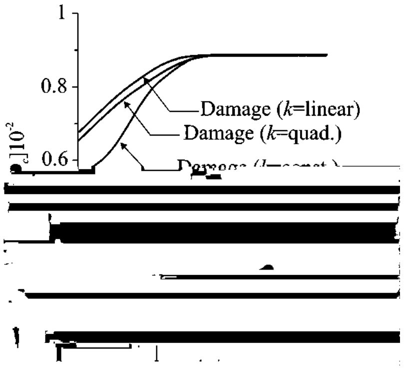

Figure 8. Evolution of damage variable at the edge of indentor

Figure 7 illustrates the evolution of pore pressure at two locations of the indentor; the centre of indentor ($r/a=0$) and the edge of indentor ($r/a=1$) corresponding to a depth of $z/a=1{\cdot}5$ within the poroelastic half-space. The damage models predict higher excess pore pressures in the porous medium which is consistent with observations by Cheng and Dusseault.[16] The evolution of the damage variable with time at the edge of indentor (/ = 1) at a depth of z/ = 0·1 is also shown in Figure 8.

6. CONCLUDING REMARKS

The classical theory of poroelasticity for a fluid saturated brittle geomaterial has been extended through computational modelling to include the influence of both damage evolution in the geomaterial fabric and alterations in the fluid transport behaviour due to damage evolution. This latter modification to the modelling is considered to be a novel development in the application of damage mechanics concepts to the study of poroelastic phenomena. The studies to date are based on plausible damage evolution laws which are derived from a limited database of experimental results. The damage evolution laws based on micromechanical considerations, on the other hand, will require considerably more analytical efforts and the incorporation of the possible influences of scale at which micromechanical processes generate microcrack evolution which manifests in the form of damage. The procedure applied in this paper is intended to capture phenomenological processes which can be modelled by appeal to experimentation. The computational modelling of damage evolution in the geomaterial fabric and the alteration in the hydraulic transport characteristics can be easily accomodated with a conventional formulation of computational modelling of transient processes in poroelastic media. The indentation problem modelled in this paper illustrates an example where damage evolution and permeability alterations can occur in zones of high local contact stresses. The modelling strictly excludes the possible development of strain localization phenomena. It is appreciated that such strain localization phenomena can contribute to both non-homogeneity and anisotropy in the permeability characteristics which merits further consideration. Also, the incorporation of such localization effects will require a consistent formulation of the computational scheme to account for scale effects, numerical stability and mesh dependency (see e.g. Reference 34). The numerical results presented in the paper illustrate the various influences of geomaterial skeletal softening and alterations in the hydraulic transport characteristics on the consolidation rate for the indentor. For the damage laws considered in this paper, the influence of hydraulic property alterations due to damage appears to have a greater influence on the time-dependent consolidation rate of the indentor.

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