Lateral Loading of a Rigid Rock Socket Embedded in a Damage-Susceptible Poroelastic Solid

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Abstract: The paper presents a computational assessment of the influence of damage on the behavior of a rigid rock socket embedde
in a fluid-saturated poroelastic solid. The iterative computational scheme takes into consideration the irreversible alteration in the flui

embedded at the surface of a damage-susceptible poroelastic half-space. In addition to the consideration of the alteration in the elasticity and hydraulic conductivity characteristics with strain, we also consider the influence of the stress-state dependency on the evolution of damage. In this latter approach, the isotropic damage evolution depends on the sense tensile/compressive of the first invariant of the effective stress tensor. Such an approximation is in keeping with common experimental observations conducted on soft rocks that damage cannot be initiated through an increase in the compressive effective confining stress that can be applied to a geomaterial. This assumption is in agreement with the experimental observations in the geological porous media with an interconnected network of pores including soft rocks and subjected to stress levels well below the failure. The studies by Katti and Desai 1995 and Park and Desai 2000, that use the disturbed state concepts indicate that intergranular bonded materials including overconsolidated clays and saturated sands with potential for exhibiting microstructural instability in the form of liquefaction, when subjected to high stress levels, can experience damage even at stress states that are compressive. The specific problem examined in the paper is illustrated in Fig. 1. The surface of the damage susceptible poroelastic medium is assumed to be free draining. The interface between the rigid rock socket and the poroelastic medium can possess pore pressure boundary conditions that correspond to either fully drained or impervious conditions. The displacements of the poroelastic medium are continuous across the rock socket–geomaterial interface. The dominant displacement of the embedded rock socket corresponds to lateral displacement at the point of application of the load $P\ t$. This load is represented by a time dependency in the form of a Heaviside step function. The time-dependent consolidation response of the rock socket is assessed in relation to its displacement at the point of application of $P\ t$.

Governing Equations

The basic investigation associated with this paper relates to the incorporation of damage mechanics within the context of the classical theory of poroelasticity proposed by Biot 1941. The evolution of stress-state-dependent isotropic damage will influence the alteration of the basic material parameters associated with the poroelastic model, in terms of the elastic modulus and the hydrausponds to rupture and irreversible deformations within the porous skeleton. In the limit when D D_C, the material response is bound to deviate from the elastic model that is adopted in the current study. It is therefore desirable to limit the range of applicability of the damage to a critical value D_C, which can be used as a normalizing parameter, against which levels of damage can be compared. In a geomaterial that experiences isotropic damage, the net stress tensor ${}^{n}_{ij}$ is related to the stress tensor ${}_{ij}$ in the undamaged state by

$${}^{n}_{ij} = \frac{{}_{ij}}{1 \bullet D} \qquad 2$$

The deformability parameters applicable to an initially isotropic elastic material, which experiences isotropic damage, can be updated by adjusting the linear elastic shear modulus by its equivalent applicable to the damaged state, i.e.

$${}^{d} = \; 1 \bullet D \qquad 3$$

In the following, we examine the mechanics of poroelastic materials that exhibit isotropic damage through the introduction of a poroelastic model with a constitutive relationship of the form

$${}_{ij} = 2\; 1 \bullet D \quad {}_{ij} + \frac{2\; 1 \bullet D}{1 \bullet 2}\; {}_{kk}\; {}_{ij} + p\; {}_{ij} \qquad 4$$

where ${}_{ij} = u_{i,j} + u_{j,i}\; /2$ = infinitesimal strain tensor; ${}_{kk} = u_{i,i}$; =linear elastic shear modulus; =Poisson’s ratio; and ${}_{ij}$=Kronecker’s delta function. Implicit in Eq. 4. is the assumption that Poisson’s ratio for the material experiencing damage is unaltered from its value applicable to the undamaged material. This assumption was put forward by Lemaitre 1984 in the strain equivalence hypothesis. In addition to the constitutive behavior defined by Eq. 4 , it is also necessary to prescribe a damage evolution criterion that can be based on either micromechanical considerations or determined through experimentation. Experimental data on measurement of damage evolution are scarce; the limited data on sandstone were examined and they propose a damage evolution criterion that is defined by

$$\frac{D}{{}_{d}} = \frac{{}_{d}}{1 + \; {}_{d}} \; 1 \bullet \frac{D}{D_C} \qquad 5$$

where ${}_{d}$=equivalent, shear strain defined by

$${}_{d} = \; e_{ij} e_{ij}\; {}^{1/2}, \quad e_{ij} = \; {}_{ij} \bullet \tfrac{1}{3}\; {}_{kk}\; {}_{ij} \qquad 6$$

and , =positive material constants. In this formulation, the normalizing damage measure is the critical damage, D_C, which is associated with the damage corresponding to a limit value of the strength of the soft rock under uniaxial compression such that the deformations cannot be treated as a reversible elastic response and should be modeled by appeal to plasticity. It should be noted that the damage evolution function defined by Eq. 5 satisfies the second law of thermodynamics. The evolution of the damage variable can be obtained by the integration of Eq. 5 between limits D_0 and D, where D_0 is the initial value of the damage variable corresponding to the intact state e.g., zero for materials in a virgin state. Integrating Eq. 5 between the limits, the evolution of D can be prescribed as follows:

$$D = D_C \bullet \; D_C \bullet D_0 \;\; 1 + \; {}_{d} \; {}^{/\, D_C} \text{á} \exp \bullet \; {}_{d}/D_C \qquad 7$$

The development of damage criteria that can account for alterations in the hydraulic conductivity during evolution of damage in saturated geomaterials, is necessary for the modeling of such phenomena in poroelastic media. Literature on the coupling between microcrack developments and permeability evolution in saturated geomaterials is primarily restricted to the experimental evaluation of the alteration in permeability of geomaterials that are subjected to triaxial stress states. Zoback and Byerlee 1975 have documented results of experiments conducted on granite and Shiping et al. 1994 give similar results for tests conducted on sandstone Fig. 2 . These studies illustrate that the fluid transport characteristics of geomaterials can be increased due to evolution of damage in porous fabric. Kiyama et al. 1996 observed similar results for the permeability evolution in granites subjected to triaxial stress states, which suggests that localization phenomena and fluid pressure-induced microfracturing could result in significant changes in the permeability in the localization zones.

Based on the experimental studies conducted by Shiping et al. 1994 , which examine the damage-induced increase in the hydraulic conductivity of sandstone due to the applied shear strains, Mahyari and Selvadurai 1998 have obtained the following relationship for the dependency of hydraulic conductivity on the equivalent shear strain ${}_{d}$, which takes the form

$k^{\text{dn accourm}}$

niques for the study of the poroelasticity is now well established and details of these advances can be found in the references cited above. The basic Galerkin procedure can be applied to convert the governing partial differential equations to their matrix equivalents applicable to a finite domain. The resulting matrix equations take the form

K C u98.099 665.4019 Tm (T)Tj /F5 1 ÞT

C^T • tH + E

block. In the finite element modeling, the interface between the rock socket and the poroelastic region does not correspond to a precise cylindrical surface; the mesh division adopted ensures that

Table 1. Comparison of Translational Stiffness of Rigid Rock Socket Subjected to Lateral Load and Elastic Behavior of Medium

	$2P_0/\ d\ _h$		
Method of analysis	L/d=1.0	L/d=2.0	L/d=4.0
Present study	6.28	9.71	11.26
Selvadurai and Rajapakse 1985	7.03	9.8	11.44

teration in the hydraulic conductivity characteristics;

3. Poroelastic response with both damage evolution and hydraulic conductivity alteration; and
4. Stress-state dependency of poroelastic response with both damage evolution and hydraulic conductivity alteration.

Numerical Results and Discussion

The idealized problem corresponds to a rigid rock socket that is embedded at the surface of a half space. In the computational modeling, however, the domain is restricted to a finite region. Therefore it is necessary to evaluate the accuracy of the finite domain used in the computational modeling in representing, approximately, a half-space region. To aid this evaluation we first examine the problem of a rigid rock socket that is embedded in an elastic half-space region. This problem has been examined by a number of investigators and a comprehensive mathematical treatment of the problem is given by Selvadurai and Rajapakse 1985. The results obtained by Selvadurai and Rajapakse 1985 for the rigid rock socket were compared with equivalent results obtained through a finite element modeling of the domain of finite extent. The results obtained, through the two schemes, for $P_0/2d\ _h$, where P_0 is the lateral load; is the linear elastic shear modulus; d is the rock socket diameter; and $_h$ is translational displacement of the head of the rock socket along the lateral load direction for those different values of L/d are shown in Table 1. The results show reasonable agreement between the analytical and the computational estimates.

The computational modeling of a rigid rock socket Fig. 1 embedded in brittle poroelastic medium susceptible to damage was conducted through the iterative finite element technique, the computational algorithm of which is shown in Fig. 3, with basic procedures that take into account damaged-induced alterations in both the elasticity, and the hydraulic conductivity characteristics. The porous skeletal material has a Poisson's ratio and the pore fluid is assumed to be nearly incompressible, i.e., $_u=0.499$. We also assume that the damage evolution is well below the levels of damage corresponding the strain levels at the peak stress state. This excludes the necessity for consideration of any strain-softening effect. The theoretical basis of the computational scheme is therefore applicable to elastic states prior to the attainment of the peak stress leading to failure or the development of strain softening postpeak. The material parameters used in the computations are those that are provided for sandstone by Shiping et al. 1994 and are as follows: E=8,300 MPa; =0.195; = =130 damage parameters; $_C$ compressive strength=30 MPa; $_T$ tensile strength=3 MPa; $D_C=0.75$ critical damage variable; $k_0=10^{-6}$ m/s; and =3.0 10^5.

The finite element discretization of the three-dimensional domain containing the laterally loaded rigid rock socket is shown in Fig. 6. The computational modeling is performed for different length to diameter L/d ratios of the rigid rock socket. The nondimensional parameter, which is used to represent the transient

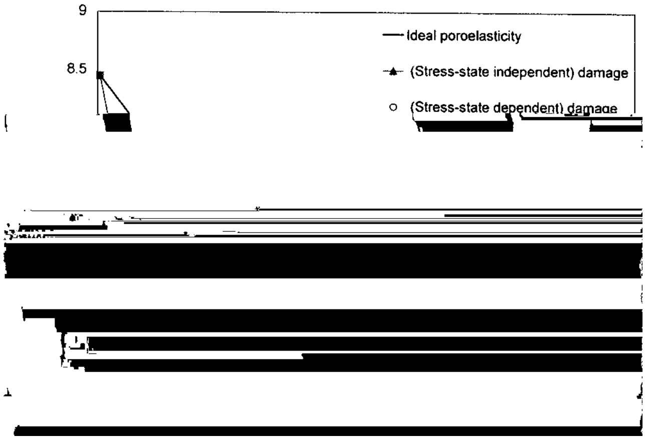


Fig. 7. Numerical results for transient translational displacement of rigid rock socket L/d=1.0 embedded in brittle poroelastic half space pervious interface

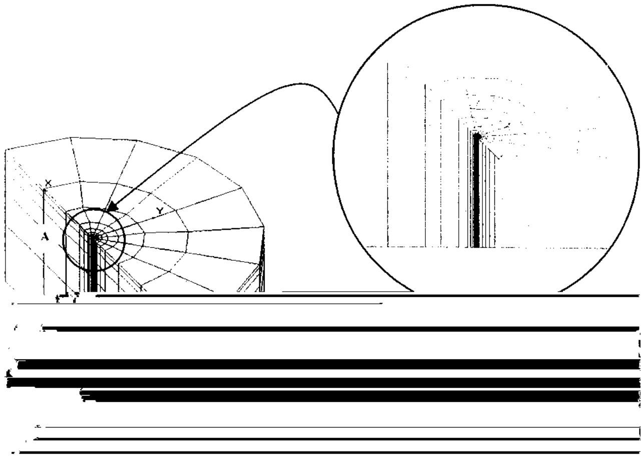


Fig. 6. Finite element discretization for rigid rock socket embedded in poroelastic half-space

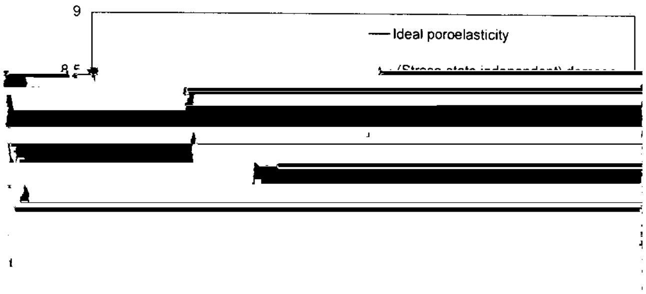


Fig. 8. Numerical results for transient translational displacement of rigid rock socket L/d=1.0 embedded in brittle poroelastic half space impervious interface

translational displacement of the rigid rock socket, is the same as

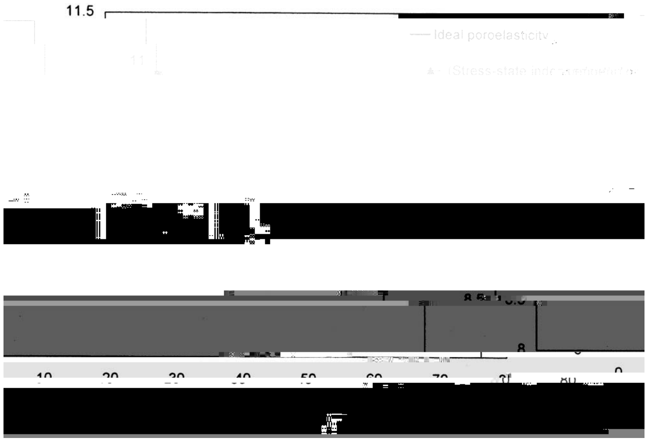


Fig. 13. Numerical results for transient translational displacement of rigid rock socket $L/d = 2.0$ embedded in brittle poroelastic half

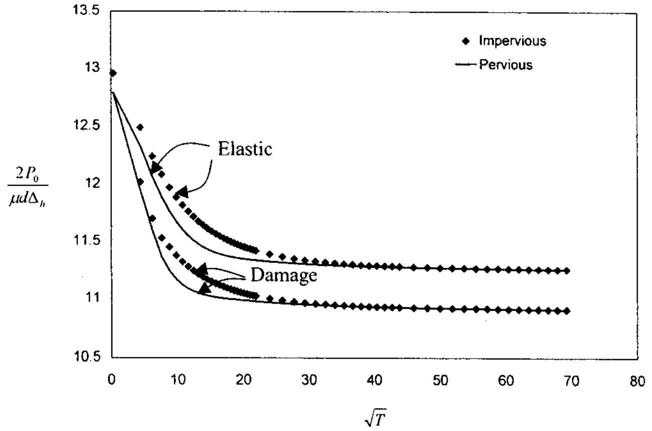
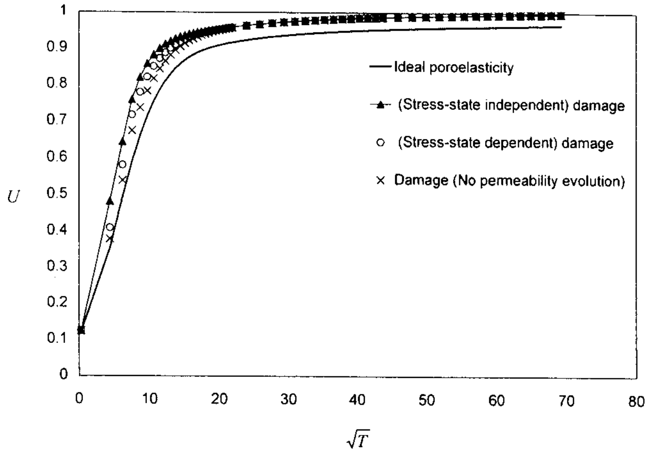


Fig. 19. Comparison of results for rigid rock socket with L/d=4.0 with either pervious or impervious interface between rock socket and poroelastic half space

interface and an impervious interface for the rock socket geometry, L/d=1.0. The influence of damage-induced alterations in the case of the impervious interface is greater, but the overall change is not significant. This is due to the highly localized response between rock socket and poroelastic medium, which extends to a region substantially smaller in comparison to the surrounding poroelastic medium. As a result any changes in the pore pressure variations at the interface region have negligible effects in comparison to variations in the bulk of the poroelastic medium. The change can be attributed to slower rate of dissipation for the case of an impervious interface and any alteration in hydraulic conductivity characteristics influences the transient response at a greater rate. Figs. 10 and 11, respectively, illustrate the degree of consolidation for the pervious and impervious pore pressure boundary conditions at the interface of the rock socket and poroelastic half space. The rate of consolidation increases where the alterations in hydraulic conductivity characteristics are taken into consideration. In addition, for the case of stress-state-dependent damage evolution, less change has been observed. Figs. 12–21 illustrate identical results applicable to rock socket dimensions defined by L/d=2.0 and 4.0.



Concluding Remarks

The classical theory of poroelasticity for a fluid saturated brittle geomaterial has been adopted to investigate the isotropic damage-induced alterations in both deformability and hydraulic conductivity parameters. An iterative finite element technique has been used to examine the influence of the isotropic damage-induced alterations in the hydraulic conductivity of the porous medium on the time-dependent response of a rigid rock socket, with different length to diameter ratios, embedded in a brittle poroelastic half space and subjected to a lateral load. Investigations have been carried out for cases involving both pervious and impervious pore pressure boundary conditions at the interface between rock socket and poroelastic medium. The numerical results presented in this paper examine both time-dependent transient translational displacement and the time-dependent degree of consolidation. The results of the computational modeling illustrate that the consideration of hydraulic conductivity alteration during the damage evolution process has a significant influence on the actual time-dependent translational displacement of the rock socket, whereas its influence on the degree of consolidation is marginal. This influence depends on the rock socket dimensions as defined by the length to diameter ratio. The larger this ratio, the greater the influence observed, and this also depends on the pore pressure boundary conditions at the interface of the rock socket and surrounding poroelastic half space. The effects are greater when the pore pressure boundary conditions correspond to an impervious interface. The dependency of the transient response on the stress state in the surrounding poroelastic half space also supports the above conclusions.

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