

Figure 4. Generalized displacements of the rigid elliptical disc anchor

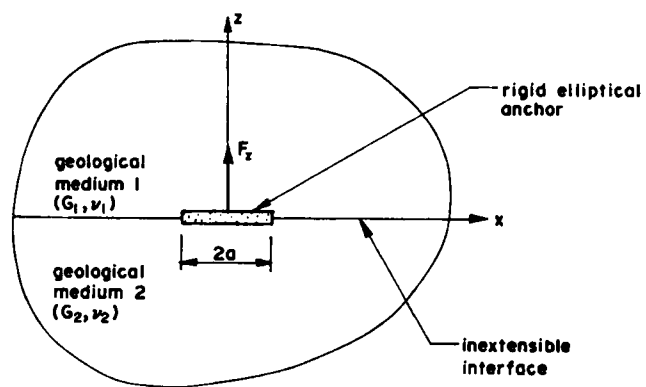
can be obtained in the form

$$\{\mathbf{u}_{S^*}\} = [\mathbf{B}]\{\Delta\} \quad (14)$$

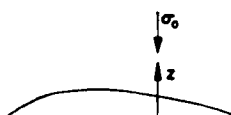
Thus (19) can be written in the form

$$\{\mathbf{F}\} = - \begin{bmatrix} c_{11} & c_{12} & 0 & 0 & 0 & 0 \\ c_{21} & c_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & c_{33} & c_{34} & 0 & 0 \\ 0 & 0 & c_{43} & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{bmatrix} \begin{bmatrix} v_1 \\ \omega_2 \\ v_2 \\ \omega_1 \\ v_3 \\ \omega_3 \end{bmatrix} \quad (22)$$

This formally completes the boundary element analysis of the problem of a rigid elliptical disc



(a) upper bound analysis







*Bounds for the axial elastic stiffness. Considering the techniques presented in the preceding*

where  $C$  is an arbitrary constant and  $e_0^2 = (a^2 - b^2)/a^2$ . The variable  $u$  is related to the ellipsoidal co-ordinate  $\xi$  by

$$\xi^2 = a^2(sn^{-2}u - 1) \quad (53)$$

$$E(u) = \int_0^u dn^2 t dt \quad (54)$$

The quantities  $sn u$ ,  $dn u$ , etc., represent the Jacobian elliptic functions<sup>13</sup> which have real and imaginary roots  $4K$  and  $2iK$ , respectively, corresponding to the moduli  $e_0$  and  $e_0^1 = b/a$ . It may also be noted that  $E(e_0)$  is the complete elliptic integral of the second kind.<sup>12</sup> Considering the boundary condition (48) and (52), it is possible to determine the constant  $C$ . Avoiding details

stiffness of the elliptical rigid anchor embedded at the bi-material interface is identical to the equivalent set developed for the axial stiffness of the elliptical anchor (equation 41)

appropriate solution (67) by virtue of the imposed constraint on  $u$  (given by (62)). Again, the

stiffness properties of the rigid elliptical anchor region. In the presentation of the numerical

variables:

$$\bar{F}_z = \frac{F_z}{4\pi a \Delta_z (G_1 + G_2) / K(e_0)} \quad (70a)$$

$$F_x = \frac{F_x}{4\pi a \Delta_x (G_1 + G_2) e_0^2 / 3 [K(e_0) - E(e_0)]} \quad (70b)$$

$$\bar{F}_x^* = \frac{F_x}{4\pi a^2 \Omega_x^* (G_1 + G_2) e_0^2 / 3 [K(e_0) - E(e_0)]} \quad (70c)$$

$$\bar{M}_y = \frac{M_y}{4\pi a^3 \Omega_y (G_1 + G_2) e_0^2 / 3 [K(e_0) - E(e_0)]} \quad (70d)$$

$$\bar{M}_z = \frac{M_z}{4\pi a^3 \Omega_z (G_1 + G_2) / K(e_0)} \quad (70e)$$

where  $K(e_0)$  and  $E(e_0)$  are complete elliptic integrals of the first and second kind. The reciprocal

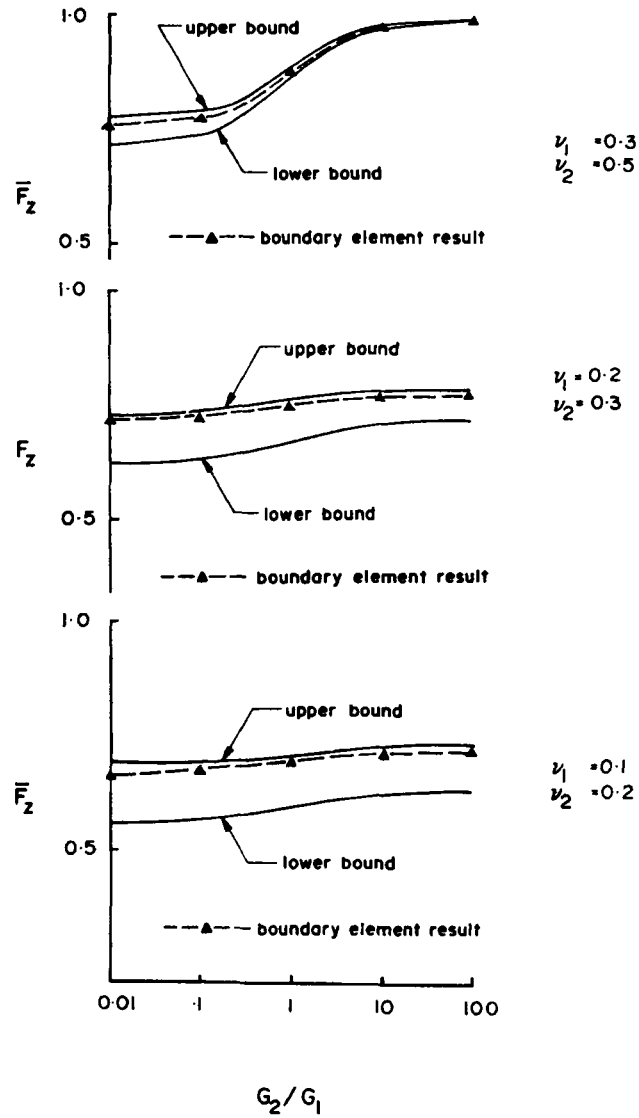


Figure 10. Axial stiffness of rigid elliptical inclusions in a bi-material medium for  $\nu_1 = 0.3$ ,  $\nu_2 = 0.5$  (top);  $\nu_1 = 0.2$ ,  $\nu_2 = 0.3$  (middle);  $\nu_1 = 0.1$ ,  $\nu_2 = 0.2$  (bottom).

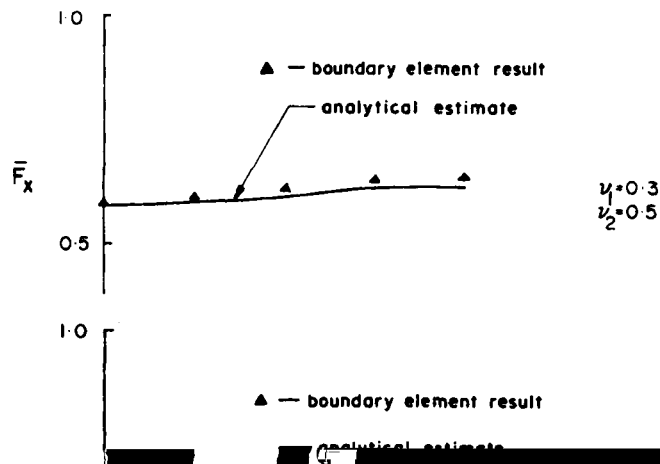


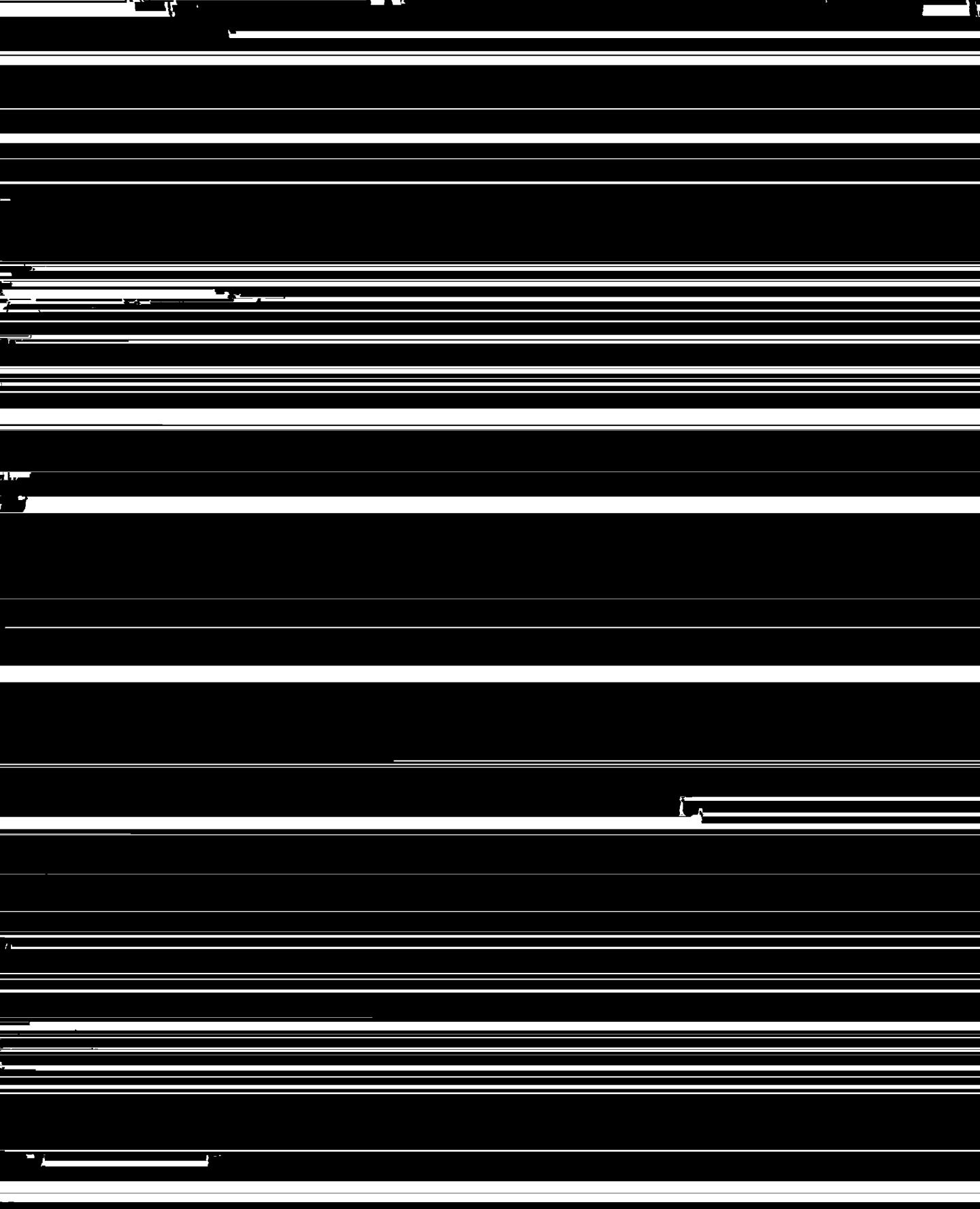
the bounds converge to a single result which agrees quite accurately with the boundary element estimate. Similar conclusions apply, in general, for the results for the non-dimensional rotational stiffness  $\bar{M}_y$ . Since the bounds for the axial stiffness are identical to the bounds for the rotational stiffness, the following relationship may be used to derive  $M_y/\Omega_y a^2$  from  $F_z/\Delta_z$ :

$$\frac{M_y}{\Omega_y a^2} = \frac{F_z K(e_0) e_0^2}{3 \Delta_z \{K(e_0) - E(e_0)\}} \quad (71)$$

Relationship (71) applies only for the bounding estimates.

Figure 11 illustrates the boundary element results for the non-dimensional translational stiffness





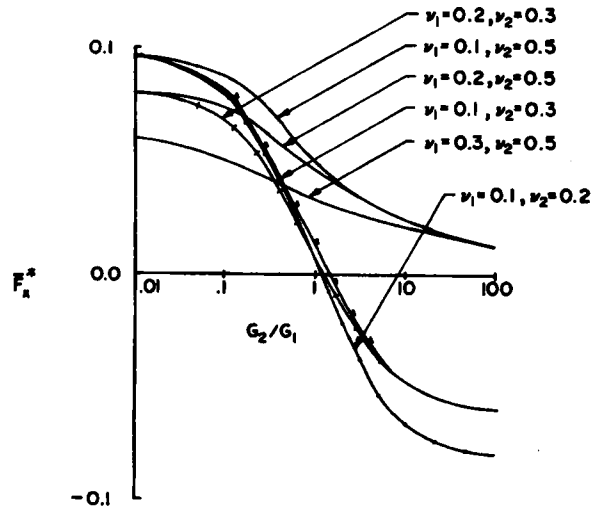


Figure 12. Coupling stiffness (rotation) due to lateral force of rigid elliptical anchor embedded in a bi-material

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