and

$$\nabla^2 \psi(r, \theta, z) = 0 \tag{2}$$

where $\nabla^2$ is Laplace's operator referred to the generalized cylindrical polar coordinate system. The displacement and stress components referred to the cylindrical polar coordinate system can be expressed in terms of $\varphi(r, \theta, z)$ and $\psi(r, \theta, z)$ in the following forms:

$$2Gu_r = -\frac{\partial^2 \varphi}{\partial r\,\partial z} + \frac{2}{r}\frac{\partial \psi}{\partial \theta} \tag{3}$$

$$2Gu_\theta = -\frac{1}{r}\frac{\partial^2 \varphi}{\partial \theta\,\partial z} - 2\frac{\partial \psi}{\partial r} \tag{4}$$

$$2Gu_z = 2(1-\nu)\nabla^2 \varphi - \frac{\partial^2 \varphi}{\partial z^2} \tag{5}$$

and

$$\sigma_{rr} = \frac{\partial}{\partial z}\left(\nu\nabla^2 - \frac{\partial^2}{\partial r^2}\right)\varphi + \frac{\partial}{\partial \theta}\left(\frac{2}{r}\frac{\partial}{\partial r} - \frac{2}{r^2}\right)\psi \tag{6}$$

$$\sigma_{\theta\theta} = \frac{\partial}{\partial z}\left(\nu\nabla^2 - \frac{1}{r}\frac{\partial}{\partial r} - \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}\right)\varphi - \frac{1}{r}\frac{\partial}{\partial \theta}\left(2\frac{\partial}{\partial r} - \frac{2}{r}\right)\psi \tag{7}$$

$$\sigma_{zz} = \frac{\partial}{\partial z}\left\{(2-\nu)\nabla^2 - \frac{\partial^2}{\partial z^2}\right\}\varphi \tag{8}$$

$$\sigma_{\theta z} = \frac{1}{r}\frac{\partial}{\partial \theta}\left\{(1-\nu)\nabla^2 - \frac{\partial^2}{\partial z^2}\right\}\varphi - \frac{\partial^2 \psi}{\partial r\,\partial z} \tag{9}$$

$$\partial \{(1 \ldots \partial^2\} \ldots \frac{1}{}\,\partial^2 \psi \quad (10)$$

force $T$ which acts in the $+ve\, x$ direction. The in-plane displacement of the rigid circular inclusion is denoted by $\delta$. From an examination of the problem,

plane $z = 0$. Therefore, we may restrict the analysis to the examination of a where $H_n[f(\xi); r]$ is the Hankel transform of order $n$ which is defined by

$$H_n[f(\xi); r] = \int_0^\infty \xi f(\xi) J_n(\xi r) \, d\xi . \tag{24}$$

We now make the assumption that as $b \to \infty$, we should recover, from the [illegible]

tion of a penny-shaped rigid inclusion which is embedded in an uncracked elastic solid. It is convenient to introduce functions $C(\xi)$ and $D(\xi)$ such that

$$A(\xi) = \frac{1}{\xi^3} \{C(\xi) + 2D(\xi)\} \tag{25}$$

$$B(\xi) = -\frac{1}{\xi^2 (1 - 2\nu)} \{C(\xi) + D(\xi)\} . \tag{26}$$

The integral Eqs. (20)–(23) can now be written as

[illegible]

to its argument. The integral Eq. (40) can be written in the form

$$f_2(u_1 b) = \frac{2c}{\pi u_1} \int_0^1 f_1(a t_1) \left\{1 + \frac{c^2 t_1^2}{u_1^2} + \frac{c^4 t_1^4}{u_1^4} + \frac{c^6 t_1^6}{u_1^6} + \frac{c^8 t_1^8}{u_1^8} + 0(c^{10})\right\} \mathrm{d}t_1; \quad u_1 > 1. \tag{56}$$

Using the third equation of (44) and the expression for $F_1(t_1)$ given by (48) in (56) we obtain the following result for $f_2(u_1 b)$:

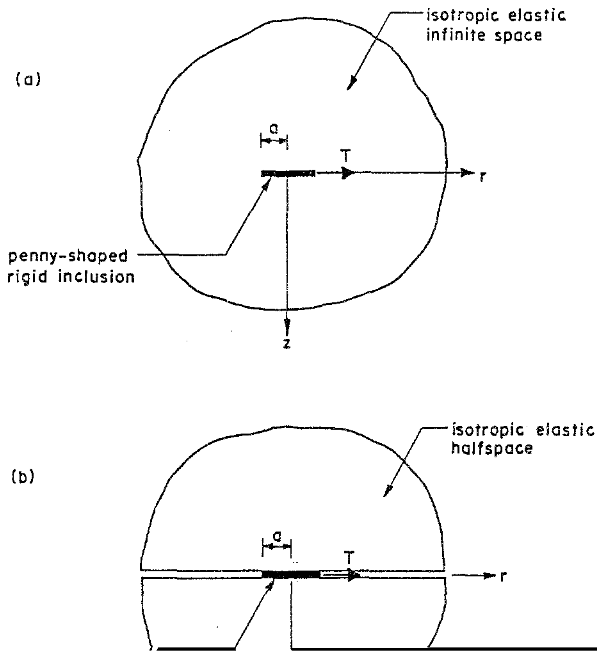
$$f_2(u_1 b) = \frac{32 G \delta (1-\nu)(1-2\nu)}{(7-8\nu)\pi^2 u_1} \cdot \left[c - 2c^2\zeta + c^3\left\{4\zeta^2 + \frac{1}{3u_1^2}\right\} - 2c^4\zeta\left\{\frac{5}{18} + 4\zeta^2 + \frac{1}{3u_1^2}\right\}\right.$$

here the accuracy of the series estimates in predicting exact solution to certain limiting cases.

In the limiting case when the radius of the externally cracked region becomes infinite (i.e., $c \to 0$), the result (53) reduces to

$$T = \frac{64\, G\, \delta\, a\, (1 - \nu)}{(7 - 8\,\nu)}. \tag{61}$$

This expression is in agreement with the results obtained, independently by Keer [11], Kassir and Sih [12] and Selvadurai [13] for the in-plane translational



stiffness of a penny-shaped rigid inclusion embedded in an isotropic elastic solid (Fig. 2a) by making use of potential function methods, ellipsoidal harmonic

function techniques and dual integral equation formulations respectively. For future reference, we note that in the particular instance, when the externally cracked region extends to the boundary of the penny-shaped rigid inclusion (i. e.,

## Appendix A

The general expressions for $m_i(t)$ $(i = 0, 1, 2, \ldots, 5)$ take the following forms:

$$m_0(t_1) = 1$$

$$m_1(t_1) = -2\zeta$$

$$m_2(t_1) = 4\zeta^2$$

$$m_3(t_1) = -4\zeta\left[\int_0^1 s\,m_2(s)\,\mathrm{d}s + \tfrac{1}{3}\int_0^1 s(t_1^2 + s^2)\,m_0(s)\,\mathrm{d}s\right]$$

$$m_4(t_1) = -4\zeta^2\left[\int_0^1 \frac{s}{3}(t_1^2 + s^2)\,m_1(s)\,\mathrm{d}s + \int_0^1 s\,m_3(s)\,\mathrm{d}s\right]$$

$$m_5(t_1) = -4\zeta\left[\tfrac{1}{5}\int_0^1 s(t_1^4 + s^4 + s^2 t_1^2)\,\mathrm{d}s + \int_0^1 s\,m_4(s)\,\mathrm{d}s + \tfrac{1}{3}\int_0^1 s(t_1^2 + s^2)\,m_2(s)\,\mathrm{d}s\right].$$

Explicit results for $m_i(t_1)$ $(i = 3, 4, 5)$ take the following forms:

$$m_3(t_1) = -4\zeta\,\{2\zeta^2 + \tfrac{1}{6}(\tfrac{2}{3} + t_1^2)\}$$

$$m_4(t_1) = 4\zeta^2\,\{\tfrac{1}{3}(t_1^2 + \tfrac{1}{2}) + 4(\tfrac{1}{12} + 2\zeta^2)\}$$

$$m_5(t_1) = -4\zeta\left[\frac{1}{10}\left(t_1^4 + \frac{t_1^2}{2} + \frac{1}{3}\right) + 4\zeta^2\left(\frac{1}{3} + 2\zeta^2\right) + \frac{2\zeta^2}{3}\left(t_1^2 + \frac{1}{2}\right)\right].$$

### References

[1] T. Mura, *Micromechanics of Defects in Solids*. Sijthoff and Noordhoff, The Netherlands, 1981.
[2] [illegible]