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THE INFLUENCE OF A BOUNDARY FRACTURE ON THE

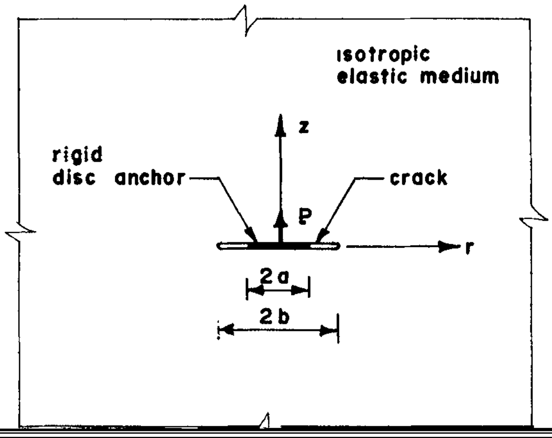
isotropic
elastic medium
z
rigid
disc anchor
crack
P
r
2a
2b

Table 1. Axial stiffness $P/8G\Delta a$ of a rigid circular punch bonded to an isotropic elastic half-space

ν	Hilbert problem approach	Hankel transform approach
0	0·549	0·546
0·1	0·597	0·596
0·2	0·657	0·658
0·3	0·736	0·737
0·4	0·841	0·842
0·5	1·000	1·000

equation formulation of the axially loaded disk anchor problem in the presence of boundary [redacted]

$$\sigma_{\theta\theta} = \frac{\partial}{\partial z}\left(\nu\nabla^2\Phi - \frac{1}{r}\frac{\partial\Phi}{\partial r}\right) \tag{11}$$

$$\sigma_{zz} = \frac{\partial}{\partial z}\left\{(2-\nu)\nabla^2 - \frac{\partial^2\Phi}{\partial z^2}\right\} \tag{12}$$

$$\sigma_{rz} = \frac{\partial}{\partial r}\left\{(1-\nu)\nabla^2\Phi - \frac{\partial^2\Phi}{\partial z^2}\right\} \tag{13}$$

where $H_n[\Omega(\xi); r]$ is the Hankel transform of order n defined by

$$H_n[\Omega(\xi); r] = \int_0^\infty \xi\Omega(\xi) J_n(\xi r)\, d\xi \tag{25}$$

By introducing the substitutions

$$H_1[M(\xi); r] = f_3(r), \quad b < r < \infty \tag{41}$$

Making use of the Hankel inversion theorem, we obtain from equations (38), (40) and (41)

$$M(\xi) = \int_0^a u f_1(u) J_1(\xi u)\,\mathrm{d}u + \int_a^b u G_1(u) J_1(\xi u)\,\mathrm{d}u + \int_b^\infty u f_3(u) J_1(\xi u)\,\mathrm{d}u \tag{42}$$

By substituting equation (42) into equation (37) and making use of the techniques outlined by

for the shear stresses in the elastic medium is

$$\sigma_{r,z}(r,0)=\frac{f_3(r)}{2(1-\nu)}+\frac{r(1-2\nu)}{2(1-\nu)}\int_r^a\frac{\phi(t)\,\mathrm{d}t}{(t^2-r^2)^{3/2}} \tag{60}$$

By making use of (48) and (50), the result (60) may be expressed in the form

$$\sigma_{r,z}(r,0)=-\frac{8G\Delta a(1-2\nu)F_3(r)}{\pi^2 r(3-4\nu)(r^2-b^2)^{1/2}}+\frac{r(1-2\nu)}{2(1-\nu)}\int_r^a\frac{\phi(t)\,\mathrm{d}t}{(t^2-r^2)^{3/2}},\quad b<r<\infty \tag{61}$$

where the function $\chi(\xi, \eta)$ is defined as

$$\chi(\xi, \eta) = \int_0^1 \left(\frac{1-c^2S^2}{1-S^2}\right)^{1/2} \frac{S^3\,\mathrm{d}S}{(1-\eta^2S^2)(1-\xi^2S^2)} \tag{67}$$

The expression for the axial stiffness of the anchor corresponding to equation (59) may now be written as

$$\begin{aligned}\frac{P}{16G\Delta} = -\frac{2(1-\nu)}{3-4\nu} &\times \left\{1 + \frac{4}{\pi^2}\frac{(1-2\nu)^2}{3-4\nu}\left[\int_0^c \left\{c\left(\frac{1-\xi^2}{c^2-\xi^2}\right)^{1/2} - 1\right\} \xi f(\xi)\,\mathrm{d}\xi \right.\right. \\ &\left.\left. + \frac{\pi}{2}c\int_c^1 \left(\frac{1-\xi^2}{\xi^2-c^2}\right)^{1/2} \frac{g(\xi)}{\xi}\,\mathrm{d}\xi\right]\right\}\end{aligned} \tag{68}$$

Similarly, the flaw shearing mode stress intensity factor at the boundary of the cracked region, equation (63), may be written as

As the cracking extends to infinity, the problem reduces to the case where the disk anchor is embedded in bonded contact with two identical half-space regions. The exact analytical solution for the axial stiffness of the disk anchor may then be obtained by simply considering the result, [redacted]

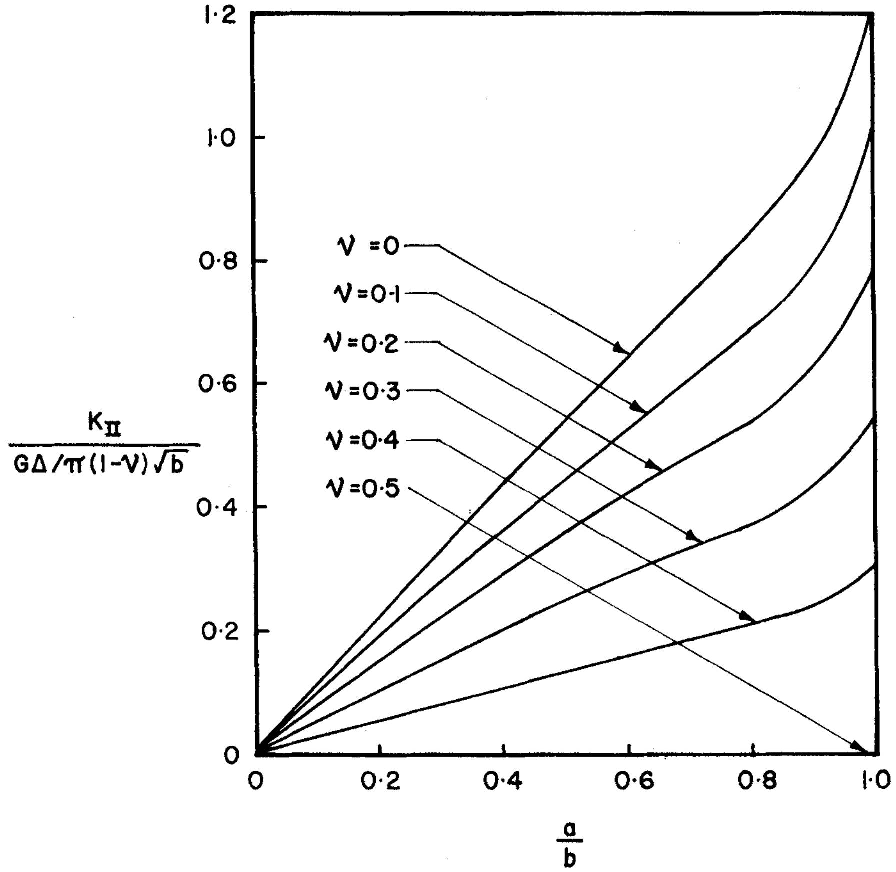

Figure 3. Variation of the flaw shearing mode stress intensity factor at the boundary of the cracked region

very accurately with the exact closed-form solutions cited in equations (71) and (72). As is evident, in the limit of material incompressibility the extent of cracking has no influence on the axial stiffness of the disk anchor. The maximum influence of the cracking on the axial stiffness of the anchor occurs when $\nu = 0$. In this case the elastostatic stiffness can be reduced by as much as approximately 25 per cent of the stiffness for the uncracked case. However, for most naturally occurring soils and rocks $\nu \in (0{\cdot}2, 0{\cdot}5)$. In this case the reduction in stiffness due to the boundary cracking is much smaller and may be considered to be of little or no practical significance. Figure 3 illustrates the manner in which the flaw shearing mode stress intensity factor at the boundary of

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